**CQF Module 3 Exam Two**



**Exotic Options Pricing Research**

**January 2024 Cohort**

This is a project on the sue of the Monte Carlo scheme to price exotic options to be completed using Python or C++. Use the expected value of the discounted payoff under the risk-neutral density

for the appropriate form of payoff (), to consider Asian and lookback options. Use the Euler-Maruyama (only) scheme for initially simulating the underlying stock price. As an initial example, the following set of sample data may be used:

|  |  |
| --- | --- |
| Today’s stock price | 100 |
| Strike | 100 |
| Time to expiry | 1Y |
| Volatility | 20% |
| Constant risk-free interest rate | 5% |

Then vary the data to see the effect on the option price.

**Evaluating the Accuracy and Efficiency of European Style Exotic Option Pricing in Markets with Dynamic Volatility Profiles**

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**Notice:**

This document encompasses comprehensive details regarding my research for this exam. It also includes essential proofs of some fundamental theorems and computation formulas. All content herein is the product of my own efforts. Information from other sources is properly cited. This document is intended solely for evaluation purposes by the CQF faculty and should not be used for any other purposes. The numerical results were all computed using Python. While this report does not contain the Python code itself, it includes explanatory notes on the development of the Python algorithms that underpin the solutions presented. These algorithms were designed following Object-Oriented Programming principles. The actual code can be accessed from "E2\_YINGJIEGUO\_CODE.zip".

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| Contents | |
| **Exam Declaration Form** |  |
| 1. **Introduction and Problem Statements** |  |
| Research Objectives | 1 |
| 1. **Monte Carlo Simulation Process Development** |  |
| * 1. Random Walk and Risk Neutrality | 2 |
| * 1. Euler Maruyama Scheme | 5 |
| * 1. Monte Carlo Simulation | 6 |
| 1. **European Style Asian and Lookback Options** |  |
| * 1. Asian Options | 7 |
| * 1. Lookback Options | 8 |
| 1. **Algorithm Design** | 10 |
| 1. **Comparative Analysis of Monte Carlo Simulated Prices** |  |
| * 1. Comparison of Prices across Different Volatilities for Asian Options | 11 |
| * 1. Comparison of Prices across Different Volatilities for Lookback Options | 12 |
| * 1. Impact of Varying Strike Prices on Option Pricing | 14 |
| * 1. Visualizing Option Dynamics: Fixed Strike Options | 14 |
| * 1. Visualizing Option Dynamics: Floating Strike Options | 17 |
| * 1. Error Analysis of Monte Carlo Simulations | 18 |
| 1. **Extension I: Analytical Solutions of Pricing Asian and Lookback Options** |  |
| * 1. Analytical Solutions for Pricing Asian Options | 19 |
| * 1. Analytical Solutions for Pricing Lookback Options | 29 |
| * 1. Comparison between Monte Carlo Simulations and Analytical Results | 32 |
| 1. **Extension II: Uncertain Volatility Modelling for Lookback Options** |  |
| * 1. Parametric Uncertain Volatility Estimation | 35 |
| * 1. Non-Parametric Uncertain Volatility Estimation | 37 |
| 1. **Conclusion** | 38 |
| 1. **References** | 39 |
| 1. **Appendices** |  |
| * Appendix A – Solutions of SDE | 40 |
| * Appendix B – Proof of Girsanov Theorem | 41 |
| * Appendix C – Convergence of to a continuous Wiener Process | 43 |
| * Appendix D – Proof of Convergence: Discrete Period Sampled Averages to continuous Sampled Averages (Asian Options) | 44 |
| * Appendix E – Covariance of Brownian Motion Increments | 46 |